Using models to do & learn mathematics: the area model

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Outcomes

Participants will:

- Explore why students struggle with math problems
- Understand the two roles that models play for students that struggle with mathematics
- Explain why the area model is a powerful model, including the types of problems and math concepts that the area model is useful for.

Why do students

struggle

with mathematics?

Try this problem

Bill has 36. He shares with himself and three friends. How much does each have?

You probably had to *stop* and *think*.

You might have thought to divide 36 by 3. You had to stop the automatic response and revert to your executive functions.

Executive Functions in Math Problem Solving

- Those elements of cognition that allow both the ***stop*** and the ***think*** parts of that wonderful habit teachers try to develop in the students with whom they work.
- Used to address novel situations

Fundamental components of EF

- *Inhibitory control*. Making an initial decision, sustaining attention, and pausing when automatic responses don't work.
- *Working memory*. Translating instructions into action plans, considering alternatives, relating one piece of information to another.
- *Cognitive flexibility*. Willingly entertaining alternative possibilities, changing your mind with new information, grasping unexpected opportunities.
  - Language mediates the process
  - Emotional panic hinders the process

What does "Stop and Think" look like when solving math problems?

A typical sequence*: *FOPS*

- *Find* the problem type.
- *Organize* the information in the problem using a model
- *Plan* to solve the problem.
- *Solve* the problem using the model.

Jitendra, Griffin, Haria, Leh, Adams, and Kaduvetoor (2005)

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This is so important because school math has

changed.

Some shifts in the Common Core Standards

- Focus on Coherence across grades
- Focus on Conceptual Understanding: seeing math as more than a set of mnemonics or discrete procedures
- Focus on Application: Using contexts to make meaning of mathematics, and using mathematics to make meaning of contexts.

The math that students are expected to *learn* has changed.

Standards for mathematical practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

The math that students are expected to *do* has changed.

Why it's so difficult to be an intervention specialist

Executive functioning

Teaching students to *stop* and *think*

- *Inhibitory control*, including initial decision, sustained attention, and pausing when automatic responses don't work
- *Working memory*. Translating instructions into action plans, considering alternatives, relating one piece of information to another
- *Cognitive flexibility*. Willingly entertaining alternative possibilities, changing your mind with new information, grasping unexpected opportunities

What students are expected to *do* and *learn*

- Make sense of problems and persevere in solving them
- Construct viable arguments
- Look for and make use of structure
- See coherence across grades
- Gain conceptual understanding
- Use contexts to make meaning of mathematics, and use mathematics to make meaning of contexts.

Using models to do & learn mathematics

Why it's so difficult to be an intervention teacher

Executive functioning

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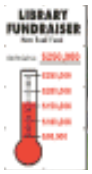
On the one hand…

Informal experiences
- Contain mathematical principles
- Context-bound
- Models <u>of</u> a situation



On the other hand…

Formal mathematics
- Potentially very general
- Far removed from context

$$\frac{3}{4}$$

Informal experiences
- Contain mathematical principles
- Context-bound
- Models <u>of</u> a situation

Formal mathematics
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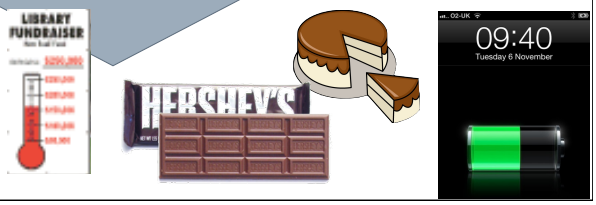
How to "connect" formal mathematics with students' informal experiences?

Informal experiences
- Contain mathematical principles
- Context-bound
- Models of a situation



"Traditional" sequence

Formal mathematics
- Potentially very general
- Far removed from context

$\frac{3}{4}$

Informal experiences
- Contain mathematical principles
- Context-bound
- Models of a situation

"Traditional" sequence

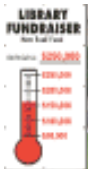
- Mathematics is disconnected from reality
- Math is seen as meaningless
- Little opportunity to participate in mathematical practices

"Discovery" sequence

Formal mathematics
- Potentially very general
- Far removed from context

$\frac{3}{4}$

Informal experiences
- Contain mathematical principles
- Context-bound
- Models <u>of</u> a situation



"Discovery" sequence

Better! But still…

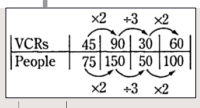
- There is a big jump from informal experiences to formal mathematics – often too big.
- Ultimately, formal mathematics is the only tool that students have to solve problems

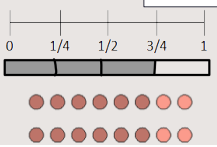
The "model and tool layer"

Formal mathematics
- Potentially very general
- Far removed from context

$\frac{3}{4}$

Models and tools
- Generalizable, but still retain contextual cues
- Models for mathematics





Informal experiences
- Contain mathematical principles
- Context-bound
- Models of a situation

Models for mathematics…

…help students *learn* formal mathematics

…are tools that students can use to *do* mathematics

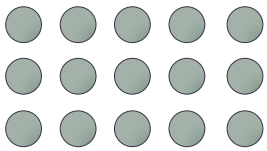
The area model

The area model

Pictorial representation of multiplication

Array model

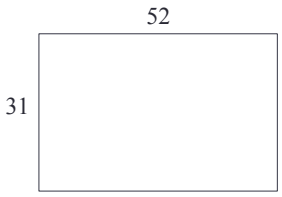
3×5



Discrete,
rows & columns

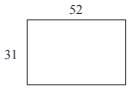
Area model

31×52



Continuous,
length & width

The area model



Helps students *learn* mathematics

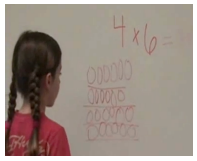
- Multiplicative reasoning
- Distributive, and commutative properties
- Multiplication and division with whole numbers, fractions, and polynomials, including traditional algorithms

Is a tool that students can use to *do* grade-level mathematics

3rd grade — 4th grade — 5th grade — 9th grade

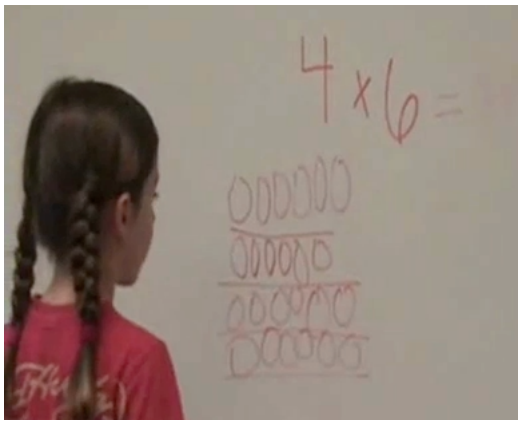
Grade-level standard (3.OA.5): Apply properties of operations as strategies to multiply and divide

Task: Find the product: $4 \times 6 = \_\_\_\_\_$



Watch the video at:
http://www.schooltube.com/video/a1033c0c8056ac32854e/

$4 \times 6 = \_\_\_\_\_$

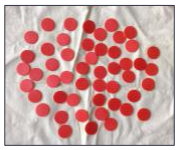


1. How is the array model helping this student ***learn*** mathematics? What does the array model reveal about multiplication?
2. How is the array model helping this student ***do*** grade-level mathematics?

3rd grade | 4th grade | 5th grade | 9th grade

Grade-level standard (3.OA.3): Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities

Task: Pasha has 50 candies to share between her and a friend. How many does each get?



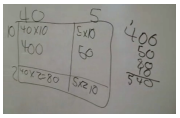
Talk with your neighbor:
How could students use an array model to help them solve this problem?

How would the array model help students ***learn*** mathematics?
What does the array model reveal about division?

3rd grade | 4th grade | 5th grade | 9th grade

Grade-level standard (4.NBT.): Multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Task: Mrs. Barton ordered 45 dozen cupcakes for the school reading celebration. How many cupcakes did she order?



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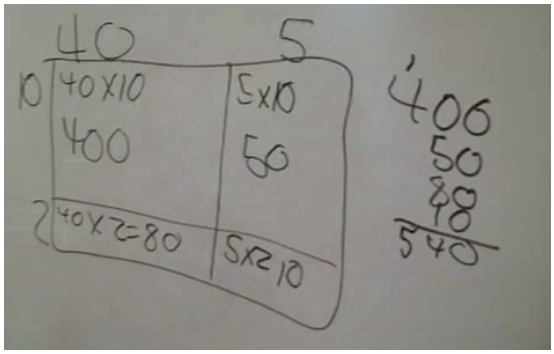
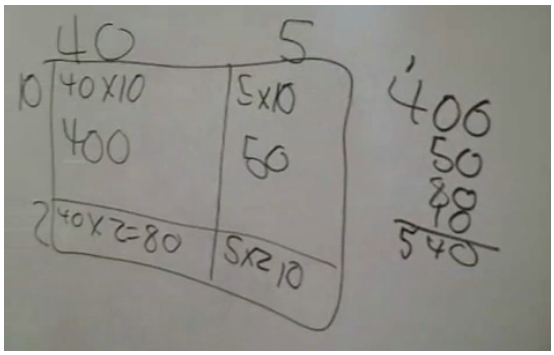
How many cupcakes are in 45 dozen?



1. How is this similar to the 4 × 6 problem in terms of the model? How is it different?

How many cupcakes are in 45 dozen?



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2. How is the area model helping this student ***do*** grade-level mathematics?

Task design continuum

Task Design					
Setting		Range of Numbers		Level of Support	Procedure for Direct Instruction
Context mimics model ←---→ Context distant from model	Concrete material ←---→ Representational ←---→ Abstract	1/2 1/3, 1/5	0-5	Max. Scaffolding ←---→ No Scaffolding	I do, You watch, We talk
		1/4, 1/8, 1/10	0-10		I do, You help, We talk
		1/6	0-20		You do, I help, We talk
		1/7, 1/9	0-100		You do, I watch, We talk
			0-1000		You do, Someone else watches, We talk
			>1000		

Fourth grade

Mrs. Barton ordered 45 dozen cupcakes for the school reading celebration. How many cupcakes did she order?

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Setting		Range of Numbers		Level of Support	Procedure for Direct Instruction
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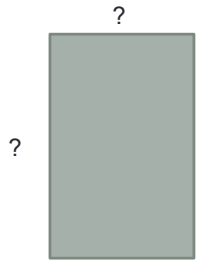
			>1000		

Fourth grade

Mrs. Barton ordered 45 dozen cupcakes for the school reading celebration. How many cupcakes did she order?

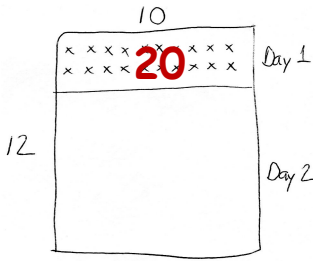
Mrs. Barton ordered 15 dozen cupcakes for the school reading celebration. She had a student set them out on a table in rows of 12 cupcakes each. How many cupcakes did the student set out?



Fourth grade

Mrs. Barton ordered 45 dozen cupcakes for the school reading celebration. How many cupcakes did she order?

Mrs. Barton is planting a garden. Her garden has room for 12 rows with 10 plants in each row. She plants 2 rows one day and plants the rest the next day. How many plants does she plant during the two days?



Third grade

$4 \times 6 = \_\_\_\_\_$

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3rd grade — 4th grade — **5th grade** — 9th grade

Grade-level standard (5.NF.6): Solve real world problems involving multiplication of fractions

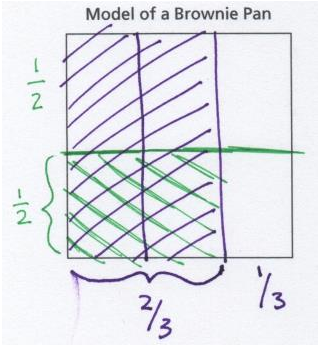
Task: All the pans of brownies are square. A pan of brownies costs $12. You can buy any fractional part of a pan of brownies and pay that fraction of $12. For example, $\frac{1}{2}$ of a pan costs $\frac{1}{2}$ of $12.

A. Mr. Williams asks to buy $\frac{1}{2}$ of a pan that is $\frac{2}{3}$ full.

1. Use a copy of the brownie pan model shown at the right. Draw a picture to show how the brownie pan might look before Mr. Williams buys his brownies.
2. Use a different colored pencil to show the part of the brownies that Mr. Williams buys. Note that Mr. Williams buys *a part of a part* of the brownie pan.
3. What fraction of a whole pan does Mr. Williams buy? What does he pay?

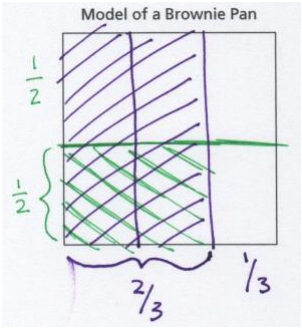
Model of a Brownie Pan

How much of a pan is 1/2 of 2/3?



1. How is this similar to the other problems in terms of the model? How is it different?

How much of a pan is 1/2 of 2/3?

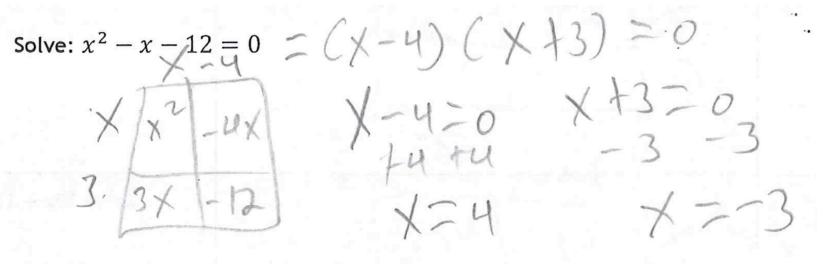


1. How does the area model help students ***learn*** mathematics?
   What does the area model reveal about multiplication of fractions?
2. How does the area model help students ***do*** grade-level mathematics?
3. Where is this problem on the continuum of instruction?
   How might you design a problem higher and lower on the continuum that helps students use the area model?

3rd grade — 4th grade — 5th grade — 9th grade

Grade-level standard (A.SSE.3a): Factor a quadratic expression to reveal the zeros of the function it defines.

Task: Solve: $x^2 - x - 12 = 0$



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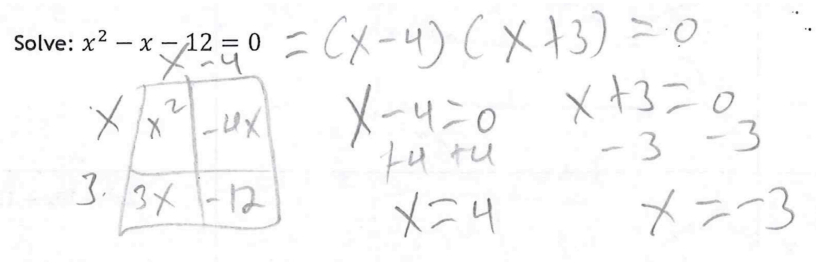


1. How is the area model helping this student ***learn*** mathematics?
   What does the area model reveal about quadratic functions and polynomials?
2. How is the area model helping this student ***do*** grade-level mathematics?
3. Where is this problem on the continuum of instruction?
   How might you design a problem higher and lower on the CoI that helps students use the area model?

Summary

$\frac{3}{4}$

Formal mathematics

Students have rich experiences that anticipate formal mathematics

Summary

$\frac{3}{4}$

Formal mathematics

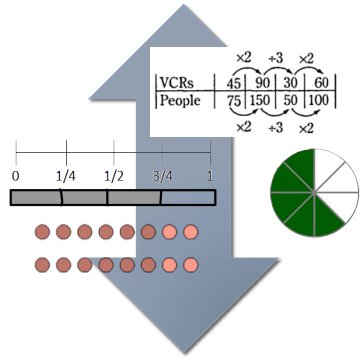
But there is still a big gap between these informal experiences and formal mathematics

Students have rich experiences that anticipate formal mathematics

Summary

$\frac{3}{4}$

Formal mathematics



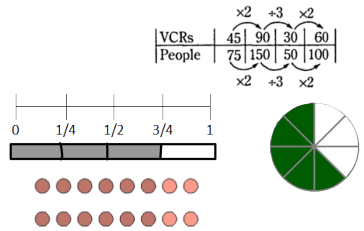
Models for learning

Students have rich experiences that anticipate formal mathematics

Summary

$\frac{3}{4}$

Formal mathematics



Models for learning

- Help students **learn** formal mathematics
- Serve as tools that students can use to **do** mathematics

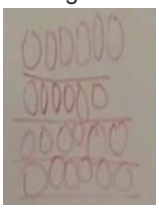
Students have rich experiences that anticipate formal mathematics

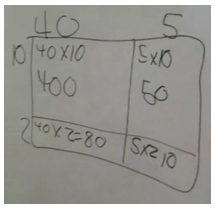
Summary :: The area model

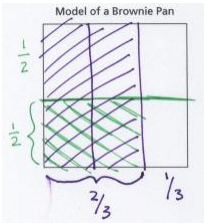
- Visual representation of multiplication
- Helps students *learn* mathematics
  - Illuminates important mathematical concepts such as distributive and commutative properties
  - Can help students understand traditional algorithms, from whole numbers to fractions to polynomials
- Is a tool that students can use to *do* grade-level mathematics
  - Allows for computational flexibility

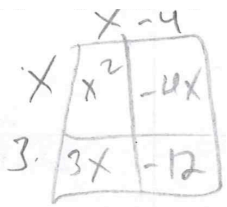
Summary :: The area model

3rd grade — 4th grade — 5th grade — 9th grade









Summary :: Task design for models

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- Slides and handouts from today
- Lots of resources for area models and other models – by teachers, for teachers

Fred – f@fapeck.com
Mark – Msemmler@CherryCreekSchools.org