Using models to do & learn mathematics: the number line

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Outcomes

Participants will:

- Explore why students struggle with math problems
- Understand the two roles that models play for students that struggle with mathematics
- Explain why the number line model is a powerful model, including the types of problems and math concepts that the number line is useful for.

Why do students

struggle

with mathematics?

Try this problem

Bill has 39. He has 12 more than Sam. How much does Sam have?

You probably had to *stop* and *think*.

You couldn't just use the automatic rule that "more than" means to add. You had to stop the automatic response and revert to your executive functions.

Executive Functions in Math Problem Solving

- Those elements of cognition that allow both the *stop* and the *think* parts of that wonderful habit teachers try to develop in the students with whom they work.
- Used to address novel situations

Fundamental components of EF

- *Inhibitory control*. Making an initial decision, sustaining attention, and pausing when automatic responses don't work.
- *Working memory*. Translating instructions into action plans, considering alternatives, relating one piece of information to another.
- *Cognitive flexibility*. Willingly entertaining alternative possibilities, changing your mind with new information, grasping unexpected opportunities.
  - Language mediates the process
  - Emotional panic hinders the process

What does "Stop and Think" look like when solving math problems?

A typical sequence*: *FOPS*

- *Find* the problem type.
- *Organize* the information in the problem using a model
- *Plan* to solve the problem.
- *Solve* the problem using the model.

Jitendra, Griffin, Haria, Leh, Adams, and Kaduvetoor (2005)

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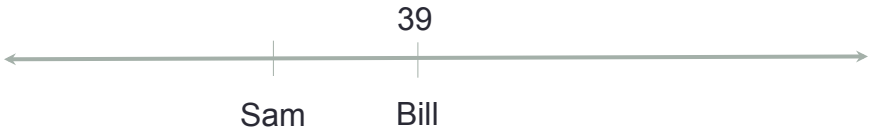
Our problem again

Bill has 39. He has 12 more than Sam. How much does Sam have?



This is so important
Because school math has
changed.

Some shifts in the Common Core Standards

- Focus on Coherence across grades
- Focus on Conceptual Understanding: seeing math as more than a set of mnemonics or discrete procedures
- Focus on Application: Using contexts to make meaning of mathematics, and using mathematics to make meaning of contexts.

The math that students are expected to *learn* has changed.

Standards for mathematical practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

The math that students are expected to *do* has changed.

Why it's so difficult to be an intervention specialist

Executive functioning

Teaching students to *stop* and *think*

- *Inhibitory control*, including initial decision, sustained attention, and pausing when automatic responses don't work
- *Working memory*. Translating instructions into action plans, considering alternatives, relating one piece of information to another
- *Cognitive flexibility*. Willingly entertaining alternative possibilities, changing your mind with new information, grasping unexpected opportunities

What students are expected to *do* and *learn*

- Make sense of problems and persevere in solving them
- Construct viable arguments
- Look for and make use of structure
- See coherence across grades
- Gain conceptual understanding
- Use contexts to make meaning of mathematics, and use mathematics to make meaning of contexts.

Using models to do & learn mathematics

Why it's so difficult to be an intervention teacher

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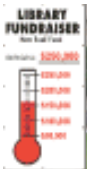
On the one hand…

Informal experiences
- Contain mathematical principles
- Context-bound
- Models <u>of</u> a situation



On the other hand…

Formal mathematics
- Potentially very general
- Far removed from context

$$\frac{3}{4}$$

Informal experiences
- Contain mathematical principles
- Context-bound
- Models of a situation

Formal mathematics
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- Far removed from context

$$\frac{3}{4}$$

How to "connect" formal mathematics with students' informal experiences?

Informal experiences
- Contain mathematical principles
- Context-bound
- Models of a situation

"Traditional" sequence

Formal mathematics
- Potentially very general
- Far removed from context

$$\frac{3}{4}$$

Informal experiences
- Contain mathematical principles
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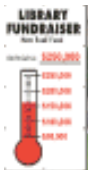


"Traditional" sequence

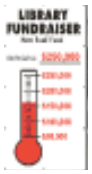
- Mathematics is disconnected from everyday reality
- Math is seen as meaningless
- Little opportunity to participate in mathematical practices

"Discovery" sequence

Formal mathematics
- Potentially very general
- Far removed from context

$$\frac{3}{4}$$

Informal experiences
- Contain mathematical principles
- Context-bound
- Models <u>of</u> a situation



"Discovery" sequence

Better! But still…

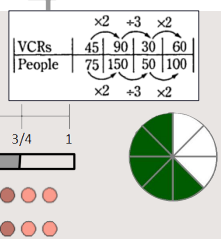
- There is a big jump from informal experiences to formal mathematics – often too big.
- Ultimately, formal mathematics is the only tool that students have to solve problems

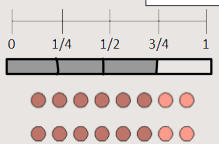
The “model and tool layer”

Formal mathematics
- Potentially very general
- Far removed from context

$$\frac{3}{4}$$

Models and tools
- Generalizable, but still retain contextual cues
- Models <u>for</u> mathematics





Informal experiences
- Contain mathematical principles
- Context-bound
- Models <u>of</u> a situation

Models <u>for</u> mathematics…

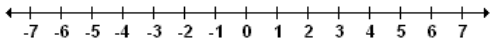
…help students *learn* mathematics

…are tools that students can use to *do* mathematics

The number line model

The number line model

Where have you seen a number line most typically used in a classroom?

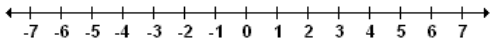


How is it typically used?

The number line model

Number lines help students both *learn* and *do* mathematics

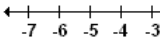
1. Number lines are linear, which is intuitive

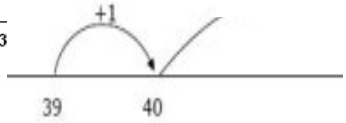


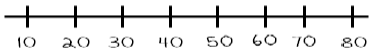
The number line model

Number lines help students both *learn* and *do* mathematics

2. Number lines are flexible and developmental







The number line model

Number lines help students both *learn* and *do* mathematics

3. Number lines reflect thinking

- 8+5=

The number line model

Helps students *learn* mathematics

- Meaning of number, number sense
- Structure of real number system (relationship between numbers, base ten, infinitely partitionable, positive and negative as inverses)
- Addition and subtraction
- Meaning of equals sign
- Meaning of algebra equations

Is a tool that students can use to *do* grade-level mathematics

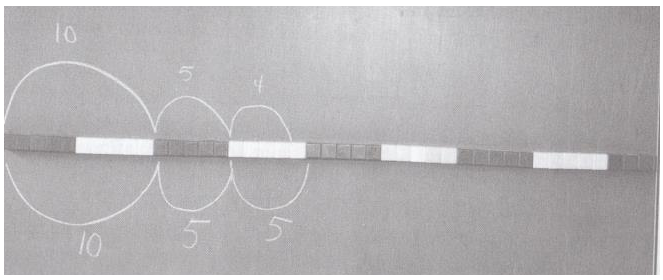
- Allows for computational flexibility

1st grade

4th grade 6th grade 8th grade

Grade-level standard (1.OA.7): Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false

Task: Is this a true equation? $5 + 4 + 10 = 10 + 5 + 5$



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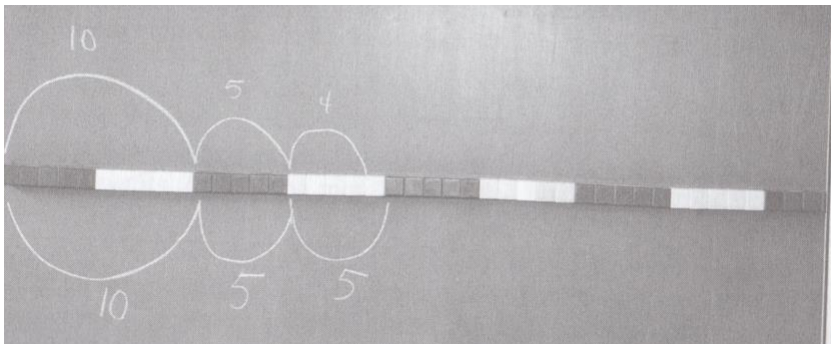
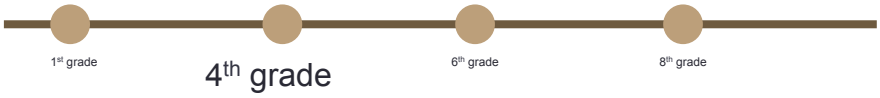
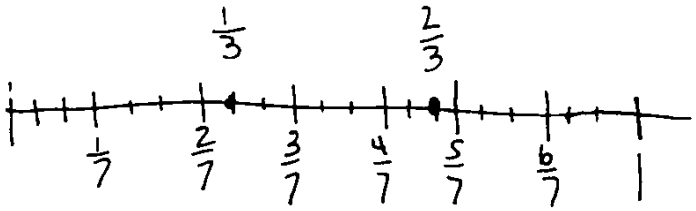


1. How is the number line helping this student ***learn*** mathematics?
   What does the number line reveal about addition, the number system, and the equals sign?
2. How is the number line helping this student ***do*** grade-level mathematics?



Grade-level standard (4.NF.2): Compare two fractions with different numerators and different denominators

Task: Which fraction is bigger: 2/3 or 5/7?



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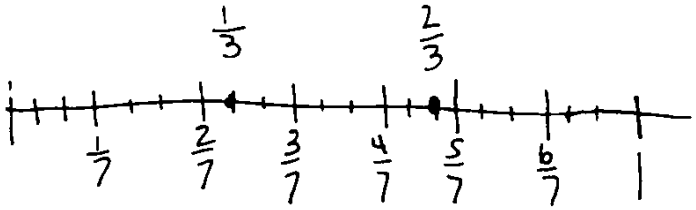

1. How is the number line helping this student ***learn*** mathematics?
   What does the number line reveal about fractions and the number system?
2. How is the number line helping this student ***do*** grade-level mathematics?

1st grade 4th grade 6th grade 8th grade

6th grade

Grade-level standard (6.NS.5): Understand that positive and negative numbers are used together to describe quantities having opposite directions or values; use positive and negative numbers to represent quantities in real-world contexts

Task: One morning the temperature is -18 degrees in Anchorage, Alaska and 75 degrees in Miami, Florida. How many degrees warmer was it in Miami than in Anchorage on that morning?

57 degrees warmer

$$\begin{array}{r} {}^{6}\cancel{7}\cancel{5}^{1}5 \\ -\ 18 \\ \hline 57 \end{array}$$

1st grade 4th grade 6th grade 8th grade

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Task: One morning the temperature is -18 degrees in Anchorage, Alaska and 75 degrees in Miami, Florida. How many degrees warmer was it in Miami than in Anchorage on that morning?

$$\begin{array}{r} {}^{1}75 \\ +\ 18 \\ \hline 93 \end{array}$$

93° warmer

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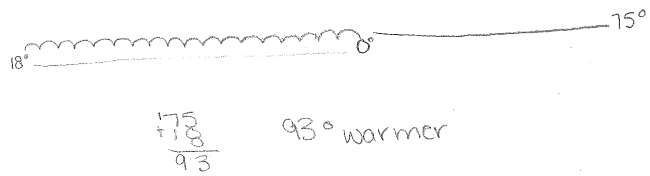

1. How is the number line helping this student ***learn*** mathematics? What does the number line reveal about positive and negative numbers?
2. How is the number line helping this student ***do*** grade-level mathematics?

Task design continuum

Task Design					
Setting		Range of Numbers		Level of Support	Procedure for Direct Instruction
Context mimics model ←---→ Context distant from model	Concrete material ←---→ Representational ←---→ Abstract	1/2 1/3, 1/5	0-5	Max. Scaffolding ←---→ No Scaffolding	I do, You watch, We talk
		1/4, 1/8, 1/10	0-10		I do, You help, We talk
		1/6	0-20		You do, I help, We talk
		1/7, 1/9	0-100		You do, I watch, We talk
			0-1000		You do, Someone else watches, We talk
			>1000		

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One morning the temperature is -18 degrees in Anchorage, Alaska and 75 degrees in Miami, Florida. How many degrees warmer was it in Miami than in Anchorage on that morning?

Context mimics model ←--→ Context distant from model

Cindy owes Fred $18. She wants to buy a new pair of shoes for $75. How much money would she need to pay Fred back and buy the shoes?

Cindy and Fred are going to race each other. The race is 75 yards long. Fred lines up at the starting line, but because Cindy is a faster runner than Fred, they agree that she will start 18 yards behind the starting line. How far will Cindy need to run?

One morning the temperature is -18 degrees in Anchorage, Alaska and 75 degrees in Miami, Florida. How many degrees warmer was it in Miami than in Anchorage on that morning?

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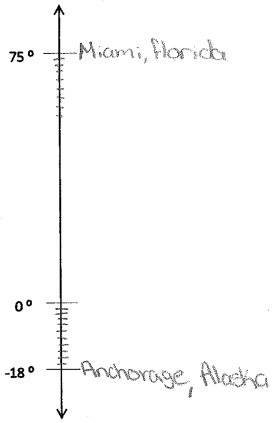
One morning the temperature is -18° in Anchorage, Alaska, and 75° in Miami, Florida. How many degrees warmer was it in Miami than in Anchorage on that morning?

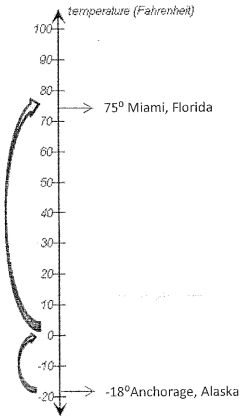


$$\begin{array}{r} 75 \\ +18 \\ \hline 93 \end{array}$$

It is 93° warmer.

Max. Scaffolding ←---→ No Scaffolding

One morning the temperature is -18^0 in Anchorage, Alaska, and 75^0 in Miami, Florida. How many degrees warmer was it in Miami than in Anchorage on that morning?



$$\begin{array}{r} 75 \\ +18 \\ \hline 93 \end{array}$$

93°F warmer because you know that it's already 75° warmer when you get to zero. Since it's in the negative you have to add 18°. which equals 93°F warmer.

Max. Scaffolding ←----------→ No Scaffolding

First grade

Is this a true equation? $5 + 4 + 10 = 10 + 5 + 5$

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Fourth grade

Which fraction is bigger: 2/3 or 5/7?

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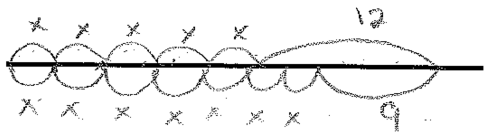
			>1000		

1st grade — 4th grade — 6th grade — 8th grade

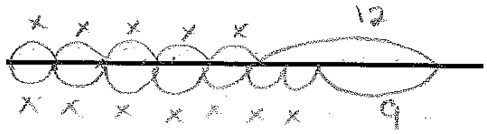
Grade-level standard (8.EE.7): Solve linear equations in one variable

Task: Solve the equation for x: $5x + 12 = 7x + 9$



5x on the top and on the bottom cancel each other out, and you're left with 2x. I subtracted 9 from both sides. 3 is left over + you divide that by 2.

$$5x + 12 = 7x + 9$$



5x on the top and on the bottom cancel each other out, and you're left with 2x. I subtracted 9 from both sides. 3 is left over + you divide that by 2.

1. How is the number line helping this student ***learn*** mathematics? What does the number line reveal about algebra equations?
2. How is the number line helping this student ***do*** grade-level mathematics?
3. Where is this problem on the continuum of instruction? How might you design a problem higher and lower on the continuum that helps students use the number line model?

Summary

$\frac{3}{4}$ Formal mathematics

Students have rich experiences that anticipate formal mathematics

Summary

$\frac{3}{4}$

Formal mathematics

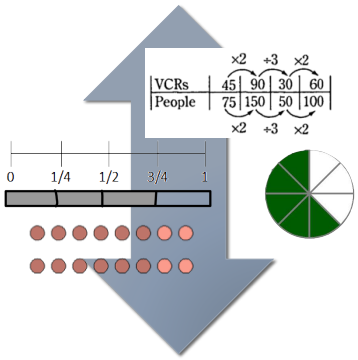
But there is still a big gap between these informal experiences and formal mathematics

Students have rich experiences that anticipate formal mathematics

Summary

$\frac{3}{4}$

Formal mathematics



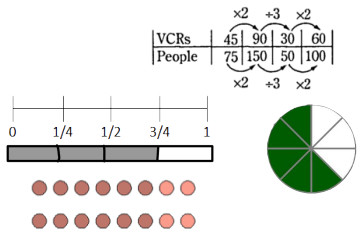
Models <u>for</u> learning

Students have rich experiences that anticipate formal mathematics

Summary

$\frac{3}{4}$

Formal mathematics



Models for learning
- Help students **learn** formal mathematics
- Serve as tools that students can use to **do** mathematics

Students have rich experiences that anticipate formal mathematics

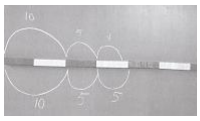
Summary :: The number line model

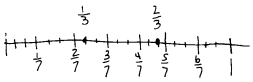
Helps students *learn* mathematics
- Meaning of number, number sense
- Structure of real number system (relationship between numbers, base ten, infinitely partitionable, positive and negative as inverses)
- Addition and subtraction
- Meaning of equals sign
- Meaning of algebra equations

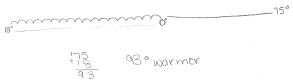
Is a tool that students can use to *do* grade-level mathematics
- Allows for computational flexibility

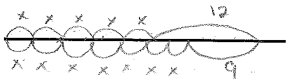
Summary :: The number line model

1st grade — 4th grade — 6th grade — 8th grade









Summary :: Task design for models

Task Design					
Setting		Range of Numbers		Level of Support	Procedure for Direct Instruction
Context mimics model ←---→ Context distant from model	Concrete material ←---→ Representational ←---→ Abstract	1/2 1/3, 1/5	0-5	Max. Scaffolding ←---→ No Scaffolding	I do, You watch, We talk
		1/4, 1/8, 1/10	0-10		I do, You help, We talk
		1/6	0-20		You do, I help, We talk
		1/7, 1/9	0-100		You do, I watch, We talk
			0-1000		You do, Someone else watches, We talk
			>1000		

Our website

www.fapeck.com/CTR

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- Slides and handouts from today
- Lots of resources for number lines and other models – by teachers, for teachers

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