Using models to do & learn mathematics: the ratio table

Courage to Risk conference | January 31, 2015

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Outcomes

Participants will:

- Explore why students struggle with math problems
- Understand the two roles that models play for students that struggle with mathematics
- Explain why the ratio table is a powerful model, including the types of problems and math concepts that the ratio table is useful for.

Why do students
struggle
with mathematics?

Try this problem

A school is raising money for a field trip. So far, they've raised $100, which is 2/5 of the total they wanted to raise. How much money is the school trying to raise?

You probably had to *stop* and *think*.

There is no obvious solution to this problem. Plus, the problem has fractions, and fractions are scary.

Executive Functions in Math Problem Solving

- Those elements of cognition that allow both the *stop* and the *think* parts of that wonderful habit teachers try to develop in the students with whom they work.
- Used to address novel situations

Fundamental components of EF

- *Inhibitory control*. Making an initial decision, sustaining attention, and pausing when automatic responses don't work.
- *Working memory*. Translating instructions into action plans, considering alternatives, relating one piece of information to another.
- *Cognitive flexibility*. Willingly entertaining alternative possibilities, changing your mind with new information, grasping unexpected opportunities.
  - Language mediates the process
  - Emotional panic hinders the process

What does "Stop and Think" look like when solving math problems?

A typical sequence*: *FOPS*

- *Find* the problem type.
- *Organize* the information in the problem using a model
- *Plan* to solve the problem.
- *Solve* the problem using the model.

Jitendra, Griffin, Haria, Leh, Adams, and Kaduvetoor (2005)

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This is so important because school math has

changed.

Some shifts in the Common Core Standards

- Focus on Coherence across grades
- Focus on Conceptual Understanding: seeing math as more than a set of mnemonics or discrete procedures
- Focus on Application: Using contexts to make meaning of mathematics, and using mathematics to make meaning of contexts.

The math that students are expected to *learn* has changed.

Standards for mathematical practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

The math that students are expected to *do* has changed.

Why it's so difficult to be an intervention specialist

Executive functioning

- *Inhibitory control*, including initial decision, sustained attention, and pausing when automatic responses don't work
- *Working memory*. Translating instructions into action plans, considering alternatives, relating one piece of information to another
- *Cognitive flexibility*. Willingly entertaining alternative possibilities, changing your mind with new information, grasping unexpected opportunities

What students are expected to *do* and *learn*

- Make sense of problems and persevere in solving them
- Construct viable arguments
- Look for and make use of structure
- See coherence across grades
- Gain conceptual understanding
- Use contexts to make meaning of mathematics, and use mathematics to make meaning of contexts.

Using models to do & learn mathematics

Why it's so difficult to be an intervention teacher

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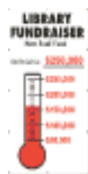
On the one hand…

Informal experiences
- Contain mathematical principles
- Context-bound
- Models of a situation




On the other hand…

Formal mathematics
- Potentially very general
- Far removed from context

$$\frac{3}{4}$$

Informal experiences
- Contain mathematical principles
- Context-bound
- Models of a situation

Formal mathematics
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$$\frac{3}{4}$$

How to "connect" formal mathematics with students' informal experiences?

Informal experiences
- Contain mathematical principles
- Context-bound
- Models <u>of</u> a situation

"Traditional" sequence

Formal mathematics
- Potentially very general
- Far removed from context

$$\frac{3}{4}$$

Informal experiences
- Contain mathematical principles
- Context-bound
- Models <u>of</u> a situation

“Traditional” sequence

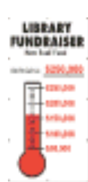
- Mathematics is disconnected from reality
- Math is seen as meaningless
- Little opportunity to participate in mathematical practices

“Discovery” sequence

Formal mathematics
- Potentially very general
- Far removed from context

$$\frac{3}{4}$$

Informal experiences
- Contain mathematical principles
- Context-bound
- Models of a situation




"Discovery" sequence

Better! But still…

- There is a big jump from informal experiences to formal mathematics – often too big.
- Ultimately, formal mathematics is the only tool that students have to solve problems

The "model and tool layer"

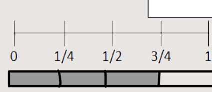
Formal mathematics
- Potentially very general
- Far removed from context

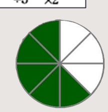
$$\frac{3}{4}$$

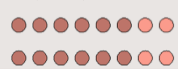
Models and tools
- Generalizable, but still retain contextual cues
- Models for mathematics

VCRs	45	90	30	60
People	75	150	50	100

×2 ÷3 ×2







Informal experiences
- Contain mathematical principles
- Context-bound
- Models of a situation

Models for mathematics…

…help students *learn* mathematics

…are tools that students can use to *do* mathematics

The

Ratio table

Consider this problem:

3 pizzas will feed nine people. How many pizzas would you need to feed 108 people?

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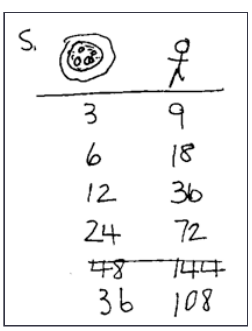
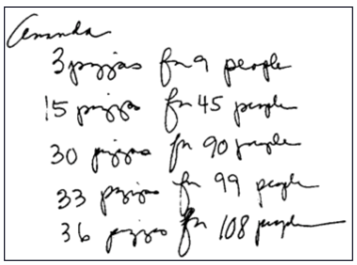


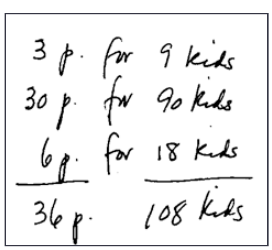
How did this student solve the problem?

3 pizzas will feed nine people. How many pizzas would you need to feed 108 people?



How about this student?

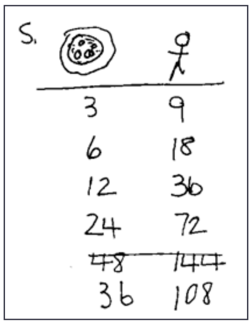
3 pizzas will feed nine people. How many pizzas would you need to feed 108 people?

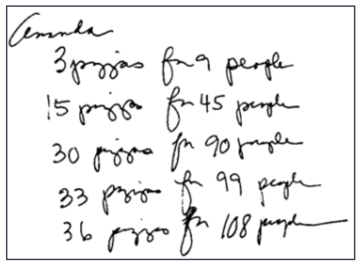


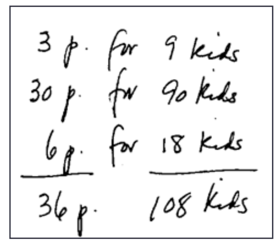
And this one?

The ratio table

3 pizzas will feed nine people. How many pizzas would you need to feed 108 people?



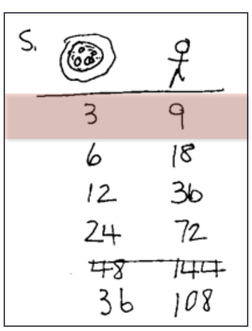


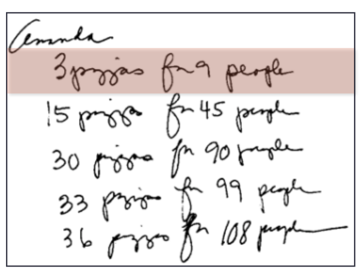


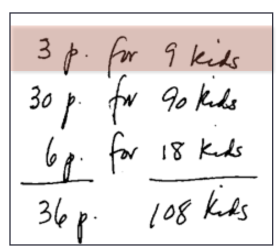
1. What are some similarities in terms of the way these three students solved the problem?

The ratio table

3 pizzas will feed nine people. How many pizzas would you need to feed 108 people?







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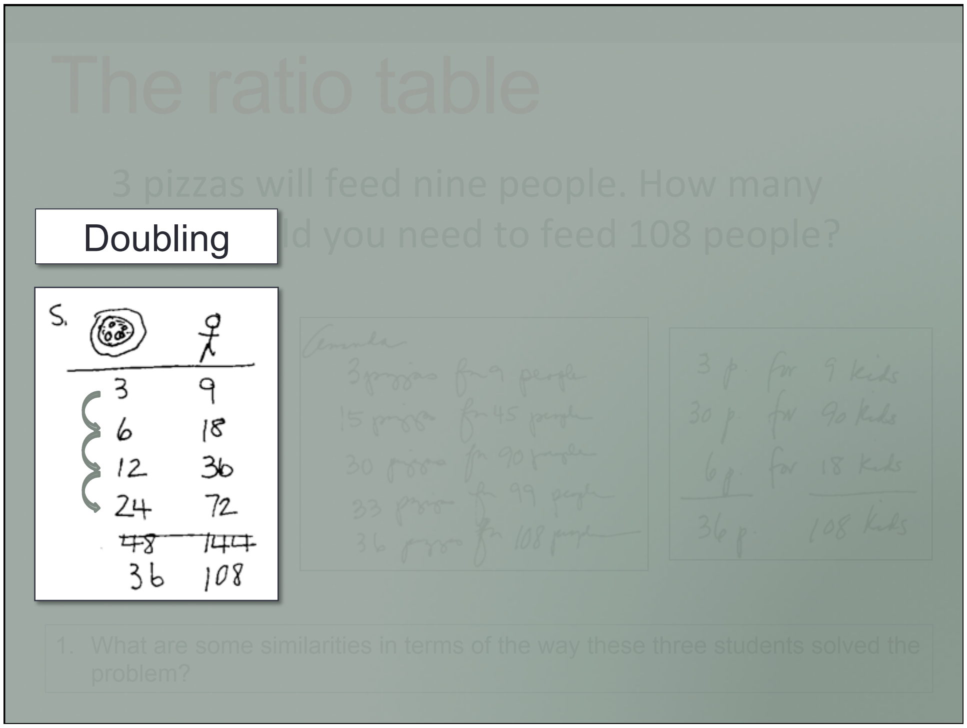
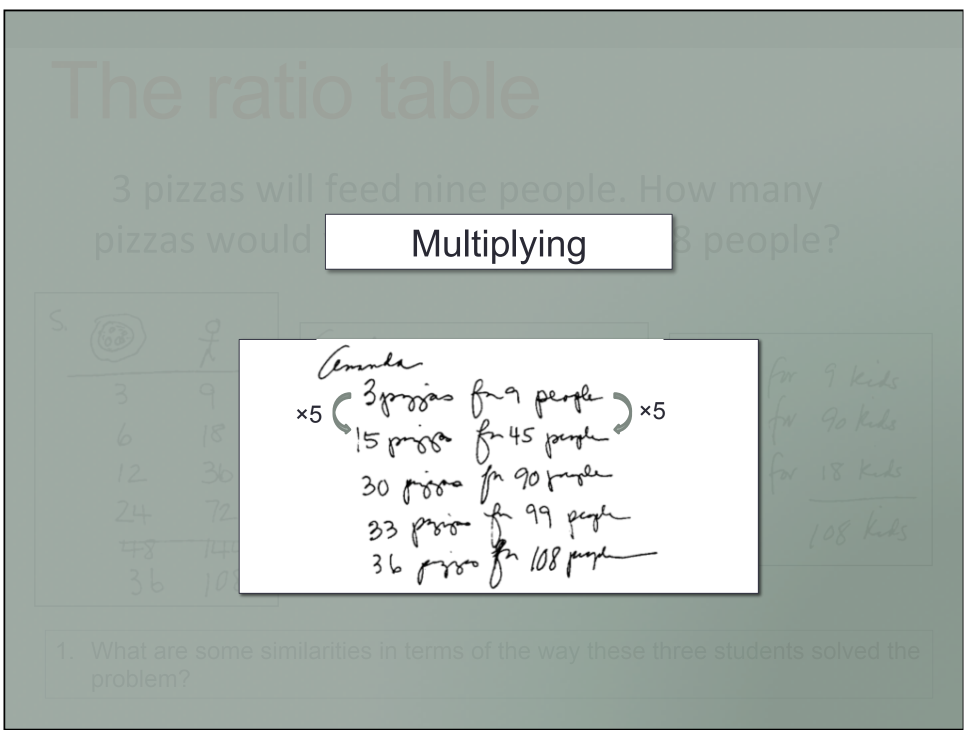
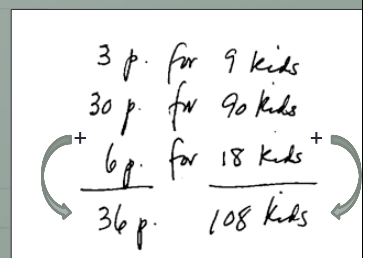

The ratio table
3 pizzas will feed nine people. How many
d you need to feed 108 people?
Doubling
S.
3 9
6 18
12 36
24 72
~~48~~ ~~144~~
36 108
1. What are some similarities in terms of the way these three students solved the problem?


The ratio table
3 pizzas will feed nine people. How many
pizzas would
3 people?
Multiplying
×5
3 pizzas for 9 people
×5
15 pizzas for 45 people
30 pizzas for 90 people
33 pizzas for 99 people
36 pizzas for 108 people
1. What are some similarities in terms of the way these three students solved the problem?

The ratio table

3 pizzas will feed nine people. How many pizzas would you need to feed 108 people?

Combining



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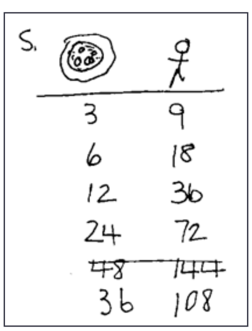

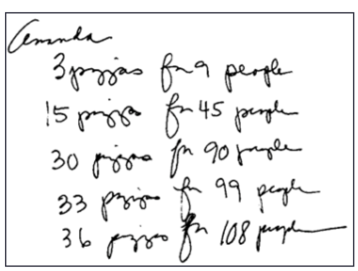

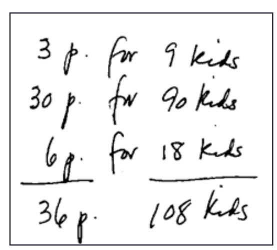

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The ratio table

Helps students *learn* mathematics

- Multiplicative reasoning
- Strategies for multiplication and division
- Relationship between multiplication and division
- Ratios and proportional reasoning
- Slope, rate of change, and linear functions

Is a tool that students can use to *do* grade-level mathematics

3rd grade — 4th grade — 6th grade — 7th grade — 9th grade

Our problem again

A school is raising money for a field trip. So far they have raised $100, which is 2/5 of the total they want to raise. How much money is the school trying to raise?

3rd grade | 4th grade | 6th grade | 7th grade | 9th grade

Grade-level standard (7.RP.1): Compute unit rates associated with ratios of fractions

A school is raising money for a field trip. So far they have raised $100, which is 2/5 of the total they want to raise. How much money is the school trying to raise?

3rd grade | 4th grade | 6th grade | 7th grade | 9th grade

Grade-level standard (7.RP.1): Compute unit rates associated with ratios of fractions

A school is raising money for a field trip. So far they have raised $100, which is 2/5 of the total they want to raise. How much money is the school trying to raise?

Fraction	1/5	2/5			
Amount		100			

Task design continuum

Task Design					
Setting		Range of Numbers		Level of Support	Procedure for Direct Instruction
Context mimics model←----→Context distant from model	Concrete material←----→Representational←----→Abstract	1/2 1/3, 1/5	0-5	Max. Scaffolding←-----------------→ No Scaffolding	I do, You watch, We talk
		1/4, 1/8, 1/10	0-10		I do, You help, We talk
		1/6	0-20		You do, I help, We talk
		1/7, 1/9	0-100		You do, I watch, We talk
			0-1000		You do, Someone else watches, We talk
			>1000		

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			>1000		

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1/5	2/5	3/5	4/5	5/5
$50	$100	$150	$200	$250

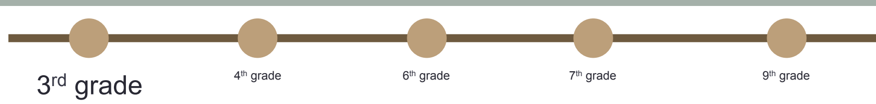
A school is raising money for a field trip. After **2** days they had raised $100, How much money will they have after **5** days?

Task Design					
Setting		Range of Numbers		Level of Support	Procedure for Direct Instruction
Context mimics model←---→Context distant from model	Concrete material←---→Representational←---→Abstract	1/2 1/3, 1/5	0-5	Max. Scaffolding←------→No Scaffolding	I do, You watch, We talk
		1/4, 1/8, 1/10	0-10		I do, You help, We talk
		1/6	0-20		You do, I help, We talk
		1/7, 1/9	0-100		You do, I watch, We talk
			0-1000		You do, Someone else watches, We talk
			>1000		

A school is raising money for a field trip. After **1** day they had raised **$50**, How much money will they have after 5 days?

Task Design

Setting		Range of Numbers		Level of Support	Procedure for Direct Instruction
Context mimics model←---→Context distant from model	Concrete material←---→Representational←---→Abstract	1/2 1/3, 1/5	0-5	Max. Scaffolding←---→No Scaffolding	I do, You watch, We talk
		1/4, 1/8, 1/10	0-10		I do, You help, We talk
		1/6	0-20		You do, I help, We talk
		1/7, 1/9	0-100		You do, I watch, We talk
			0-1000		You do, Someone else watches, We talk
			>1000		



Grade-level standard (3.OA.5): Apply properties of operations as strategies to multiply and divide

Task: Find the product: 5 * 8 = _____

3rd grade 4th grade 6th grade 7th grade 9th grade

Grade-level standard (3.OA.5): Apply properties of operations as strategies to multiply and divide

Task: Find the product: 5 * 8 = _____

Strategy: skip counting / repeated addition

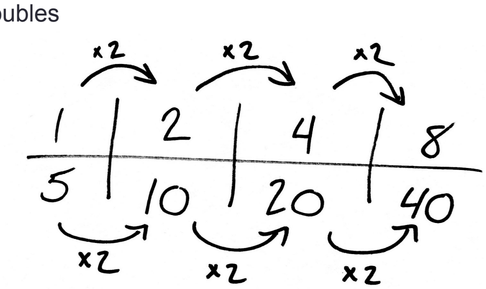
1	2	3	4	5	6	7	8
5	10	15	20	25	30	35	40

3rd grade 4th grade 6th grade 7th grade 9th grade

Grade-level standard (3.OA.5): Apply properties of operations as strategies to multiply and divide

Task: Find the product: 5 * 8 = _____

Strategy: Use doubles



3rd grade | 4th grade | 6th grade | 7th grade | 9th grade

Grade-level standard (3.OA.5): Apply properties of operations as strategies to multiply and divide

Task: Find the product: 5 * 8 = _____

Strategy: Known facts

1	5	2	8
5	25	10	40

5 * 8 = _____

Strategy:
Build up, skip count

1	2	3	4	5	6	7	8
5	10	15	20	25	30	35	40

Strategy:
Use doubles

1	2	4	8
5	10	20	40

Strategy:
Use known facts

1	5	2	8
5	25	10	40

1. How does the ratio table help students ***learn*** mathematics?
   What does the ratio table reveal about multiplication?
2. How does the ratio table help students ***do*** grade-level mathematics?

3rd grade | **4th grade** | 6th grade | 7th grade | 9th grade

Grade-level standard (4.OA.3): Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted.

Task: Each week, a farmer sells her fruit at the market. There are 149 apples left in the bottom of the crate. The farmer must put them into boxes of 12 apples each. How many more boxes does she need?

3rd grade | **4th grade** | 6th grade | 7th grade | 9th grade

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$$\begin{array}{r|l} & 12 \\ \hline 12 & 149 \\ & \underline{120} \\ & 29 \\ & \underline{24} \\ & 5 \end{array}$$

SHE NEEDS 12 R 5 BOXES.

3rd grade | **4th grade** | 6th grade | 7th grade | 9th grade

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$$
\begin{array}{r}
12.41 \\
12 \overline{)\,149.00} \\
\underline{120} \\
29 \\
\underline{24} \\
50 \\
\underline{48} \\
20 \\
\underline{12} \; \ldots \\
8
\end{array}
$$

SHE NEEDS
12.41 BOXES

3rd grade | **4th grade** | 6th grade | 7th grade | 9th grade

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BOXES	1	10	11	12	13
APPLES	12	120	132	144	156

12 BOXES IS NOT ENOUGH.
SHE NEEDS 13 BOXES.

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12 BOXES IS NOT ENOUGH.
SHE NEEDS 13 BOXES.

1. How does the ratio table help students ***learn*** mathematics?
   What does the ratio table reveal about the relationship between multiplication and division?
2. How does the ratio table help students ***do*** grade-level mathematics?

3rd grade — 4th grade — 6th grade — 7th grade — 9th grade

Grade-level standard (6.RP.3b):Solve unit rate problems including those involving unit pricing and constant speed

Task: Ms. Margo runs six miles every day. On average, it takes her 54 minutes to run six miles. At this rate, how long will it take her to run an 11 mile race?

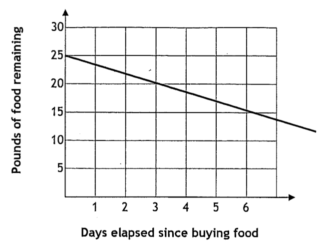
Talk with your neighbor:
How could students use a ratio table to help them solve this problem?

How would the ratio table help students ***learn*** mathematics?
What does the ratio table reveal about rates and ratios?

3rd grade 4th grade 6th grade 7th grade 9th grade

Grade-level standard (F-LE.5): Interpret the parameters in a linear function in terms of a context.

I buy big bags of food for my dog. After I buy a bag, I keep track of how much food I have left using a graph, as shown below



a. When the days change by 3, the pounds of food change by 5

b. When the days change by 1, the pounds of food change by 1.6

c. What is the rate of change in the graph? *(Use correct units)* − 1.6 lbs per day

d. How many pounds of food are in the bag when I buy it? 25 lbs.

e. Write an **equation** that I can use to predict the pounds of food after any number of days:

$y = -1.6x + 25$

3rd grade 4th grade 6th grade 7th grade 9th grade

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e. Write an equation that I can use to predict the pounds of food after any number of days:

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Summary

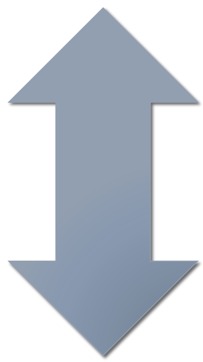
$\frac{3}{4}$ Formal mathematics

Students have rich experiences that anticipate formal mathematics

Summary

$\frac{3}{4}$

Formal mathematics



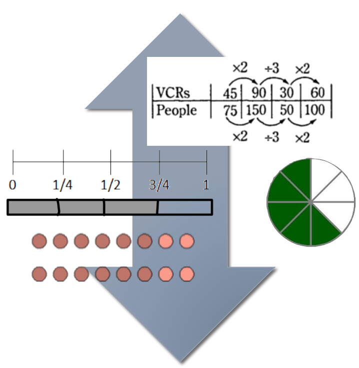
But there is still a big gap between these informal experiences and formal mathematics

Students have rich experiences that anticipate formal mathematics

Summary

$\frac{3}{4}$

Formal mathematics



Models for learning

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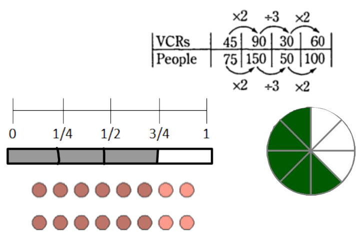


Models for learning

- Help students **learn** formal mathematics
- Serve as tools that students can use to **do** mathematics

Students have rich experiences that anticipate formal mathematics

Summary :: The ratio table

BOXES	1	10	11	12	13
APPLES	12	120	132	144	156

Helps students *learn* mathematics

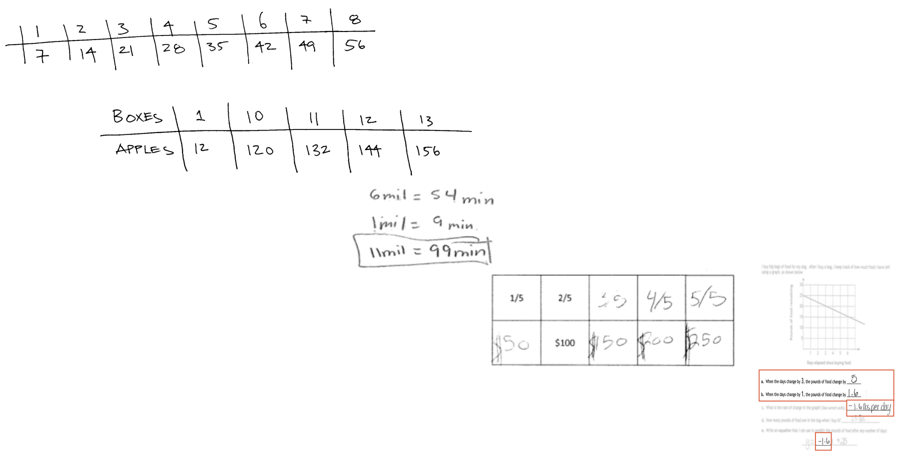
- Promotes mental math strategies in a context and through a structure that supports the development of mathematical understanding
- Develops and nurtures understanding of fractions as ratios, and fraction equivalency
- Makes salient relationships between and comparisons of quantities

Is a tool that students can use to *do* grade-level mathematics

- Allows for computational flexibility

Summary :: The ratio table

3rd grade — 4th grade — 6th grade — 7th grade — 9th grade



Summary :: Task design for models

Task Design					
Setting		Range of Numbers		Level of Support	Procedure for Direct Instruction
Context mimics model←---→Context distant from model	Concrete material←---→Representational←---→Abstract	1/2 1/3, 1/5	0-5	Max. Scaffolding←------→No Scaffolding	I do, You watch, We talk
		1/4, 1/8, 1/10	0-10		I do, You help, We talk
		1/6	0-20		You do, I help, We talk
		1/7, 1/9	0-100		You do, I watch, We talk
			0-1000		You do, Someone else watches, We talk
			>1000		

Our website

www.fapeck.com/CTR

Username: couragetorisk
Password: couragetorisk

- Slides and handouts from today
- Lots of resources for ratio table and other models – by teachers, for teachers

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